OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4724

Core Mathematics 4

Monday **23 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

 [Turn over

1 Simplify $\frac{x^3 - 3x^2}{x^2 - 9}$. [3]

2 Given that $\sin y = xy + x^2$, find $\frac{dy}{dx}$ in terms of x and y. [5]

3 (i) Find the quotient and the remainder when $3x^3 - 2x^2 + x + 7$ is divided by $x^2 - 2x + 5$. [4]

(ii) Hence, or otherwise, determine the values of the constants a and b such that, when $3x^3 - 2x^2 + ax + b$ is divided by $x^2 - 2x + 5$, there is no remainder. [2]

4 (i) Use integration by parts to find $\int x \sec^2 x \, dx$. [4]

(ii) Hence find $\int x \tan^2 x \, dx$. [3]

5 A curve is given parametrically by the equations $x = t^2$, $y = 2t$.

(i) Find $\frac{dy}{dx}$ in terms of t, giving your answer in its simplest form. [2]

(ii) Show that the equation of the tangent to the curve at $(p^2, 2p)$ is

$$py = x + p^2.$$ [2]

(iii) Find the coordinates of the point where the tangent at $(9, 6)$ meets the tangent at $(25, -10)$. [4]

6 (i) Show that the substitution $x = \sin^2 \theta$ transforms $\int \sqrt{\frac{x}{1-x}} \, dx$ to $\int 2\sin^2 \theta \, d\theta$. [4]

(ii) Hence find $\int_0^1 \sqrt{\frac{x}{1-x}} \, dx$. [5]

7 The expression $\frac{11 + 8x}{(2-x)(1+x)^2}$ is denoted by $f(x)$.

(i) Express $f(x)$ in the form $\frac{A}{2-x} + \frac{B}{1+x} + \frac{C}{(1+x)^2}$, where A, B and C are constants. [5]

(ii) Given that $|x| < 1$, find the first 3 terms in the expansion of $f(x)$ in ascending powers of x. [5]

8 **(i)** Solve the differential equation

$$\frac{dy}{dx} = \frac{2-x}{y-3},$$

giving the particular solution that satisfies the condition $y = 4$ when $x = 5$. [5]

(ii) Show that this particular solution can be expressed in the form

$$(x-a)^2 + (y-b)^2 = k,$$

where the values of the constants a, b and k are to be stated. [3]

(iii) Hence sketch the graph of the particular solution, indicating clearly its main features. [3]

9 Two lines have vector equations

$$\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix} + t\begin{pmatrix} -8 \\ 1 \\ -2 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} -2 \\ a \\ -2 \end{pmatrix} + s\begin{pmatrix} -9 \\ 2 \\ -5 \end{pmatrix},$$

where a is a constant.

(i) Calculate the acute angle between the lines. [5]

(ii) Given that these two lines intersect, find a and the point of intersection. [8]

1		Attempt to factorise numerator and denominator	M1		
		num = $xx(x-3)$ or denom = $(x-3)(x+3)$	A1		Not num = $x(x^2-3x)$
		Final answer = $\frac{x^2}{x+3}$ [Not $\frac{xx}{x+3}$]	A1	3	Do not ignore further cancellation.
2		$\frac{d}{dx}(\sin y)=\cos y.\frac{dy}{dx}$	B1		
		$\frac{d}{dx}(xy)=x\frac{dy}{dx}+y$ s.o.i.	B1		[SR: If xy taken to LHS, accept $-x\frac{dy}{dx}+y$ as s.o.i.]
		$\cos y.\frac{dy}{dx}=x\frac{dy}{dx}+y+2x$ AEF	B1		
		[If written as $\frac{dy}{dx}=\cos y\frac{dy}{dx}=x\frac{dy}{dx}+y+2x$, accept for prev B1 but not for following marks if the $\frac{dy}{dx}$ is used]			
		$f(x,y)\frac{dy}{dx}=g(x,y)$	M1		Regrouping provided > one $\frac{dy}{dx}$ term
		$\frac{y+2x}{\cos y-x}$ or $-\frac{y+2x}{x-\cos y}$ or $\frac{-2x-y}{x-\cos y}$	A1	5	ISW Answer could imply M1
3	(i)	Quotient = $3x+\ldots$	B1		For correct leading term in quotient
		For evidence of correct division process	M1		Or for cubic $\equiv(x^2-2x+5)(gx+h)\ (+\ldots)$
		$3x+4$	A1		For correct quotient
		$-6x-13$	A1	4	For correct remainder ISW
	(ii)	$a=7$	B1√		Follow through If rem in (i) is $Px+Q$,
		$b=20$	B1√	2	then B1√ for $a=1-P$ and B1√ for $b=7-Q$
		[SR: If B0+B0, award B1√ for $a=1+P$ AND $b=7+Q$; also SR B1 for $a=20, b=7$]			
4	(i)	Parts using correct split of $u=x, \frac{dv}{dx}=\sec^2 x$	M1		1st stage result of form $f(x)+/-\int g(x)dx$
		$x\tan x-\int\tan x\,dx$	A1		Correct 1st stage
		$\int\tan x\,dx=-\ln\cos x$ or $\ln\sec x$	B1		
		$x\tan x+\ln\cos x+c$ or $x\tan x-\ln\sec x+c$	A1	4	
	(ii)	$\tan^2 x=+/-\sec^2 x+/-1$	M1		or $\sec^2 x=+/-1+/-\tan^2 x$
		$\int x\sec^2 x\,dx-\int x\,dx$ s.o.i.	A1		Correct 1st stage
		$x\tan x+\ln\cos x-\frac{1}{2}x^2+c$	A1√	3	f.t. their answer to part (i) $-\frac{1}{2}x^2$

Q	Part	Answer	Mark	Total	Guidance
5	(i)	$\frac{dy}{dx}=\frac{dy}{dt}/\frac{dx}{dt}$	M1		Used, not just quoted
		$\frac{1}{t}$ or t^{-1}	A1	2	Not $\frac{2}{2t}$ as final answer
		SR: M1 for Cart conv, finding $\frac{dy}{dx}$ & ans involv t + A1			M1 is attempt only, accuracy not involved
	(ii)	Finding equation of tangent (using p or t)	M1		
		$py = x + p^2$	A1	2	AG; p essential; at least 1 line inter working
	(iii)	$(25,-10) \Rightarrow p=-5$ or $-5y = x+25$ seen	B1		$5y = x+25$ seen $\Rightarrow$ B0
		Substitution of their values of p into given tgt eqn	M1		Producing 2 equations
		Solving the 2 equations simultaneously	M1		
		$(-15,-2)$ $x=-15, y=-2$	A1	4	Common wrong ans $(15,8) \Rightarrow$ B0, M2, A0
6	(i)	Attempt to connect $dx, d\theta$	M1		But not $dx = d\theta$
		$dx = 2\sin\theta\cos\theta\, d\theta$	A1		AEF
		$\sqrt{\frac{x}{1-x}} = \frac{\sin\theta}{\cos\theta}$	B1		Ignore any references to $\pm$.
		Reduction to $\int 2\sin^2\theta\, d\theta$	A1	4	AG WWW
	(ii)	$\sin^2\theta = k(+/-1+/-\cos 2\theta)$	M1		Attempt to change $(2)\sin^2\theta$ into $f(\cos 2\theta)$
		$2\sin^2\theta = 1-\cos 2\theta$	A1		Correct attempt
		$\int \cos 2\theta\, d\theta = \frac{1}{2}\sin 2\theta$	B1		Seen anywhere in this part
		Attempting to change limits	M1		<u>Or</u> Attempting to resubstitute; Accept degrees
		$\frac{1}{2}\pi$	A1	5	
		<u>Alternatively</u> Parts once & use			
		$\cos^2\theta = 1-\sin^2\theta$	(M2)		Instead of the M1 A1 B1
		$\frac{1}{2}(\theta - \sin\theta\cos\theta)$	(A1)		Then the final M1 A1 for use of limits
7	(i)	$A = 3$	B1		For correct value stated
		$C = 1$	B1		For correct value stated
		$11+8x \equiv A(1+x)^2 + B(2-x)(1+x) + C(2-x)$	M1		AEF; any suitable identity
		e.g. $A-B=0, 2A+B-C=8, A+2B+2C=11$	A1		For any correct (f.t.) equation involving B
		$B = 3$	A1	5	
	(ii)	$\left(1-\frac{x}{2}\right)^{-1} = 1+\frac{x}{2}+\frac{x^2}{4}+\dots$	B1		s.o.i.
		$(1+x)^{-1} = 1-x+x^2-\dots$	B1		s.o.i.
		$(1+x)^{-2} = 1-2x \quad + 3x^2 - \dots$	B1,B1		s.o.i.
		Expansion $= \frac{11}{2}-\frac{17}{4}x+\frac{51}{8}x^2+\dots$	B1	5	CAO. No f.t. for wrong A and/or B and/or C

SR(1) If partial fractions not used but product of $(11+8x)(2-x)^{-1}(1+x)^{-2}$ attempted, then

B1 for $\left(1-\frac{x}{2}\right)^{-1}=1+\frac{x}{2}+\frac{x^2}{4}+\dots$

B1,B1 for $(1+x)^{-2}=1-2x+\dots+3x^2+\dots$

B1,B1 for $\frac{11}{2}-\frac{17}{4}x+\dots+\frac{51}{8}x^2+\dots$

SR(2) If partial fractions not used denominator multiplied out, then

B1 for denom $=2+3x(+0x^2)+\dots$

B1 for $\left(1+\frac{3x}{2}\right)^{-1}=1-\frac{3x}{2}+\frac{9x^2}{4}+\dots$

B1,B1,B1 for $\frac{11}{2}\dots-\frac{17}{4}x\dots+\frac{51}{8}x^2+\dots$

N.B. In both SR, if final expansion given B0, ------------------*allow SR B1 for* $22-17x+51/2\,x^2$

8	(i)	$\int(y-3)\,dy=\int(2-x)\,dx$ or equiv	M1		For separation & integration of both sides
		$\frac{1}{2}y^2-3y=2x-\frac{1}{2}x^2$	A1		or $\frac{1}{2}(y-3)^2=-\frac{1}{2}(x-2)^2$
		For an arbitrary const on one/both sides	*B1		} (or + M2 for equiv statement using limits)
		Substituting $(x,y)=(5,4)$ or (4,5) & finding 'c'	dep*M1		}
		$\frac{1}{2}y^2-3y=-\frac{1}{2}x^2+2x-\frac{3}{2}$ AEF ISW	A1	5	or $\frac{1}{2}(y-3)^2=-\frac{1}{2}(x-2)^2+5$ AEF
	(ii)	Attempt to clear fracts (if nec) & compl square	M1		
		$a=2, b=3, k=10$	A2	3	For all 3; SR: A1 for 1 or 2 correct
	(iii)	Circle clearly indicated in a sketch	B1		
		Centre $(2,3)$ or their (a,b)	B1√		
		Radius $\sqrt{10}$ or their $\sqrt{k}$	B1√	3	√ provided $k>0$

9	(i)	Using $\begin{pmatrix}-8\\1\\-2\end{pmatrix}$ and $\begin{pmatrix}-9\\2\\-5\end{pmatrix}$ as the relevant vectors	M1		i.e. correct direction vectors
		Using $\cos\theta=\frac{a.b}{\lvert a\rvert\lvert b\rvert}$ AEF for any 2 vectors	M1		Accept $\cos\theta=\left\lvert\frac{a.b}{\lvert a\rvert\lvert b\rvert}\right\rvert$
		Method for scalar product of any 2 vectors	M1		
		Method for finding magnitude of any vector	M1		
		15° (15.38...), 0.268 rad	A1	5	
	(ii)	Produce (at least) 2 of the 3 eqns in t and s	M1		e.g. $4-8t=-2-9s$, $-6-2t=-2-5s$
		Solve the (x) and (z) equations	M1		
		$t=3$ or $s=2$	A1		for first value found
		$s=2$ or $t=3$ f.t.	A1√		for second value found
		Substituting their (t,s) into (y) equation	M1		
		$a=1$	A1		
		Substituting their t into l_1 or their (s,a)			

into l_2	M1		
$\begin{pmatrix}-20\\5\\-12\end{pmatrix}$	A1	8	Any format but not $\begin{pmatrix}\ \\ \ \\ \ \end{pmatrix}+\begin{pmatrix}\ \\ \ \\ \ \end{pmatrix}$